OFF ROAD TRANSPORTATION COST CALCULATIONS FOR GROUND BASED FOREST HARVESTING SYSTEMS

NILS EGI SØVDE
Norwegian Forest and Landscape Institute, Ås, Norway. Molde University College, Molde, Norway

Abstract. Ground based systems are the main approach used for off-road timber transportation throughout the world. Estimates of terrain transportation costs are required for several forest planning problems and for assessment of harvesting contracts and forest land values. Methods for these calculations can be categorized into two groups. Methods based on average transportation distance predate computers, are analytical, and based on manual calculations. Network methods are based on weighted graph representations and are solved with graph methods. Here, the two categories are compared and linked. Analytical methods in the literature have been limited to flat terrain and including detail is difficult. The network method can be extended to include uneven terrain or detailed input data.

Keywords: terrain transportation, terrain transportation cost, forest planning, forest operations, forest harvesting

1 Introduction

Ground based systems are the dominant systems used for off road timber transportation throughout the world, either using forwarders or skidders. A harvesting operation consists of several steps. The trees have to be felled, limbed, cross cut and transported to roadside. The terrain transportation cost (TTC) is a significant part of the total harvesting cost. However, there has not been much focus on the cost calculations of off road driving in the research lately. Such calculations are required for e.g. planning of forest operations, forest planning in general or environmental planning, but seldom discussed in detail. Computational power, remote sensing techniques and optimization techniques have significantly improved the past 25 years, and a revisiting of this topic seems appropriate.

The TTC can be calculated analytically or numerically. Early methods that predate computers are in general analytical and based on average skidding distance (ASD), sometimes referred to as average yarding distance. Although such ASD-methods also can be numerical, computers and numerical methods can handle more complex and detailed models. Such methods will in this work be referred to as the network method and are the main focus of this work, as ASD-methods have been discussed elsewhere (e.g. Sundberg and Silversides 1988, Greulich 2003). In a real world case there are several factors that may influence the TTC. Detours increasing the skidding distance may be necessary due to e.g. steep slopes, rock outcrops, soil types or environmental values. Varying volume density may also affect the actual average skidding distance. Another issue is that skidding time may be varying due to terrain and soil type. Nurminen et al. (2006) found driving speeds from 14.5 m min$^{-1}$ to 87.3 m min$^{-1}$ (loaded and empty), but did not relate this to terrain types.

In this work, the ASD-method and the network method are compared and their limitations are analyzed in light of how they can be developed further. A forest parcel is a forest area which is meaningful to consider as a whole. Reasons can be equal site index, uniform or uniformly aged forest or simply that the parcel is a suitable silvicultural or computational treatment unit. A forest parcel may thus be a stand, a forest compartment, a grid point (or mesh) or other units used in model formulations. It is noteworthy that e.g. a mathematical model in general will be valid for different sizes of forest parcels, but the parcel size has impact on how the model parameters should be calculated.

2 Early methods based on average skidding distance

The calculation of harvesting and forwarding costs in forestry literature has been limited by the technology and techniques of each era. Advances in maps and surveying techniques, and in software and hardware have
improved calculations. Early approaches predate computers and were designed for calculation by humans. Greulich (2003) describes some of the early literature referring to the ASD.

Matthews (1942) treats ASD-estimates analytically. His work is a commonly cited reference for harvesting cost calculations. It describes cost calculations and optimization for varying cases (e.g. skidding and yarding, uphill and downhill, different road and landing layouts) and problems (e.g. road and landing location, choice of equipment). In this paper the focus is on TTC, and the method for calculating TTC described by Matthews (1942) will be referred to as the Matthews’ method. The Matthews’ method is simple in the sense that it is designed for hand calculations and relies on the geometric shape of the forest parcel under consideration. The mean unit TTC for a parcel, $\bar{c}$, is given by

$$\bar{c} = c_d d_c, \quad (1)$$

where $c_d$ is the mean unit distance dependent cost and $d_c$ is Matthews’ estimate of the ASD. The calculations of $d_c$ vary according to the assumptions made. If all the wood is assumed to be transported to one landing, $d_c = d(x_c, y_c)$ where the function $d(x, y) = \sqrt{(x-x_1)^2 + (y-y_1)^2}$ is the distance from $(x, y)$ to a landing $(x_1, y_1)$. If continuous landings are used, $d_c$ is found by dividing the parcel in smaller parts according to the shape of the parcel.

Suddarth and Herrick (1964) derived another estimate of ASD by integrating the function $d(x, y)$ across the parcel. If $A_p$ is the area of the parcel $p$, the ASD is given by

$$\text{ASD} = \frac{1}{A_p} \int \int_p d(x, y) \, dA. \quad (2)$$

The ASD is in general not the same as the $d_c$ in Equation (1). By replacing $d_c$ in Equation (1) with the ASD, $\bar{c}$ is given by

$$\bar{c} = c_d \cdot \text{ASD}. \quad (3)$$

The total TTC of harvesting a parcel is

$$c = \bar{c} \cdot V, \quad (4)$$

where $V$ is the timber volume of the parcel.

The integral in Equation (2) can be formulated for any parcel shape. Analytical solutions have been reported for some shapes (e.g. Suddarth and Herrick 1964, Peters 1978), but the derivations are in general cumbersome or lengthy or both. Equation (2) has also been further extended for side slope (Greulich 1980, 1987, 1989), for rectilinear thinnings (Greulich 1994a) and continuous landings (Greulich 1994b,c). The basis for TTC in the above references is Equation (3), but Greulich (1987) formalized the calculations further by introducing regression analysis. The harvest of an area was assumed to consist of a number of turns with a distance $\rho$ to the landing, and the ASD is the expected value of the random variable $\rho$. The calculations were further refined by Greulich (1991), who calculated the expected yarding cost to be

$$E\{YC\} = \beta_0 + \beta_1 w E\{\rho\} + \beta_2 w^2 E\{\rho^2\}. \quad (5)$$

The right hand side of Equation (5) is a truncated power series of order 2, approximating an unknown function. The latter two terms can be considered the expected TTC. Likewise, Equations (3) can be considered the TTC part of a truncated power series of order 1 for the same unknown function, and thus, Equation (5) will be more accurate.

The $w$ in Equation (5) is a wander factor. An extraction trail rarely follows the straight line to a landing, and throughout the forestry literature a wander factor ($w$) has been used for correcting the relation between TTC and ASD. Although von Segebaden (1964) included the wander factor and is frequently credited the invention, the concept was mentioned earlier by Hughes (1930). Usually, it is assumed that the harvest area is divided in parcels with uniform wander factor.

Another approach for corrections to skidding distance is to include obstacles when calculating TTC. This approach was used by Gibson and Rodenberg (1975), who found the skidding distance as the distance of line segments avoiding the obstacle. A similar approach was used by Perkins and Lynn (1979), who included a sub-origin some distance from the landing. The ASD was calculated either to the landing or the sub-origin, depending on the layout, and in the latter case the distance from the sub-origin to the landing was included in the ASD.

3 The network method

Models of terrain transportation based on ASD are in general analytical and possible to do by hand calculations. Such methods suits well to productivity studies based on work cycles. Likewise, ASD-methods are well adapted to forest parcels, the commonly used unit both in forestry and forest research. Most forest planning problem formulations require that input parameters (e.g. TTC) are supplied by experts. Tan (1992) introduced the network method for TTC calculations. The forest was modeled as a weighted graph, i.e. a regular network of grid points independent of forest parcels. Instead of calculating the ASD of each grid point (the ASD-method), the wood was assumed to be transported to one of the neighboring grid points and recursively through grid point until a landing was reached. For a network, there is a large number of possible paths between two grid points, but finding the cheapest one is referred to as the shortest path problem.
More precisely, the problem can be defined as follows. Let the terrain be given by a weighted graph \( G = (V, E) \), where each vertex (grid point) \( v_i \in V \) represents a point in the terrain. The edges \( E \) link each vertex with its neighbors, and the unit cost of transport between the neighbors \( v_i \) and \( v_j \) is \( c(v_i, v_j) \). A path from vertex \( v_0 \) to vertex \( v_n \) is given by \( p = (v_0, v_1, \ldots, v_n) \), and the unit cost of transporting timber on this path is the sum of its constituent edges, given by Equation (6).

\[
c(p) = \sum_{i=1}^{n} c(v_{i-1}, v_i)
\]

The mean unit TTC of grid point \( v_j \) can be found by minimizing Equation (7), i.e. the shortest/cheapest path from \( v_j \) to a vertex \( v_0 \) that is a landing.

\[
c_j = \begin{cases} 
\min \{ c(p) : v_j \xrightarrow{p} v_0 \} & \text{if there is a path from } v_j \text{ to } v_0 \\
\infty & \text{otherwise}
\end{cases}
\]

This problem formulation can be solved fast by standard combinatorial mathematics (e.g. by Dijkstra’s shortest path algorithm (Dijkstra 1959)).

The total TTC of harvesting a parcel is given by

\[
c = \sum_j V_j c_j
\]

where \( V_j \) is the timber volume at grid point \( i \). There are some studies in the literature using the shortest path formulation given by Equation (6)–(7). Although the method is largely the same, there are some variations of how the cost \( c(v_i, v_j) \) of transport to a neighbor (Equation (6)) is calculated. Tan (1992) calculated the cost as a function of distance and terrain class. Contreras and Chung (2007) used a skidding cost derived from the regression model by Han and Renzie (2005), including distance and slope gradient uphill or downhill. In a more recent paper, Contreras and Chung (2011) refined the skidding model to also include a maximum roll limit. Chung et al. (2008) used a cost based on distance. Sovde et al. (2013) used a cost based on distance and penalties for roll and pitch.

One advantage of the network method is that as long as the distance between neighboring grid points is small, there is no need for introducing a wander factor.

4 Discussion

Although the network method has been in use for more than 20 years, the method has not been analyzed in depth. The topics of the cited literature are: road planning (Tan 1992, Chung et al. 2008), landing location (Contreras and Chung 2007) and extraction trail layout (Contreras and Chung 2011, Sovde et al. 2013). As the calculation of TTC is only a part of the studied problems, the network method is not discussed much.

4.1 The impact of uneven terrain

Let \( f(x, y) \) be a function of the TTC per area, incorporating all the irregularities that may influence the TTC and also the timber volume. Without loss of generality, the parcel \( p \) is assumed to be rectangular. The parcel can be partitioned into \( n \times m \) sub-rectangles in the \( x \) and \( y \) direction. The TTC of a parcel is given by the sum of the cost of harvesting each sub-rectangle,

\[
\hat{c} = \sum_{k=1}^{n} \sum_{l=1}^{m} f(x^*_k, y^*_l) \Delta A,
\]

where \( (x^*, y^*) \) is the center point of each sub-rectangle. By increasing the number of sub-rectangles, the double sum approaches the integral.

\[
c = \lim_{n,m \to \infty} \sum_{k=1}^{n} \sum_{l=1}^{m} f(x^*_k, y^*_l) \Delta A
\]

Equation (4) follows directly from Equation (11) if \( f(x, y) = V \cdot c_d / \Delta A \cdot d(x, y) \). Finding the analytical function \( f(x, y) \) is not straightforward. The timber from a point \( (x, y) \) can be transported along many extraction trails, maybe also to different landings. One possibility is that the value of \( f(x, y) \) is the cost of using the cheapest extraction trail to the point \( (x, y) \). The ASD-method assumes that the skidding distance is the straight line to a landing. If there are obstacles, and the trail follows a known curve \( (x(t), y(t)) \), the length of the curve can be found by integrating along the curve. Also, finding the shortest curve of all the possible extraction trail curves is possible. However, if the driving speed varies, the problem of finding the fastest path turns into the problem of the brachistochrone, a more difficult problem of variational calculus. The interested reader may find more details in Troutman (1996). Whether the integral in Equation (11) can be solved easily is a matter of the function \( f(x, y) \) and the shape of the parcel.

Equation (8) is the same as Equation (9) if the sums and indices are rearranged and \( f(x^*_k, y^*_l) \Delta A = V_j c_j \). If \( c_j \) in Equation (7) is a good estimate of \( f(x, y) \), the problem of finding the TTC in uneven terrain can readily be solved with the network method.

4.2 Input data

A model formulation or a method such as the ASD-method or the network method relies on the input data. There are numerous ground based harvesting systems operating around the world and the
productivity varies. Traditionally, productivity studies in forestry are based on time studies. A harvesting operation is observed and the operation at hand is partitioned into part operations. Typically, the harvesting operation is repetitive and the data is analyzed as cycles. Time studies are time consuming and has some limitations, though. There is a limit to how much information a person can register, as well as to how detailed the data can be. Although data such as position or terrain can be registered or maybe assessed afterwards, a study relies on the predefined areas of interest. Recent productivity studies of terrain transport (e.g. Han and Renzie 2005, Nurminen et al. 2006) still report cycle times. Such data are suitable for the ASD-method, but not directly applicable for the network method. Cycle times are average values where the observed distances are larger than the raster size used in some of the later publications using the network method and the estimates of driving speed may be smoothed.

An interesting question is whether \( c_j \) in Equation (7) is a good estimate of \( f(x,y) \). This is in fact an open research question. Neither Contreras and Chung (2011) nor Søvde et al. (2013) found studies on how driving speed is affected by the micro terrain. Hopefully, such studies will appear soon. Sensors (e.g. accelerometers, gyros and gps) are available at budget prices, and high accuracy DTMs and inventories are widely used in forestry. Some other data that may influence the TTC (e.g. soil data) is not available at the same level of accuracy, but nevertheless, the prospects for future improvements are good.

### 4.3 Computational complexity

A model is a simplification of the real world, and may be arbitrarily close to the real world. In addition, some models are intractable by computers (Garey and Johnson 1979), and can not be solved to optimality for large problem instances. The computational complexity of the Matthews’ method is in general \( \mathcal{O}(n) \). If an integral for the ASD-method can be calculated, this method is also \( \mathcal{O}(n) \). In contrast, if the network method is implemented using Dijkstra’s algorithm with Fibonacci heaps, the computational complexity is \( \mathcal{O}(n \log(n)) \) (Fredman and Tarjan 1987). This may limit the problem instance size solvable by the method.

In most publications, the TTC is needed as parameters in other models. However, it is possible to include analytical TTC calculations in larger models. Greulich (2012) found the near-optimal location of two landings using a combination of continuous and numerical techniques.

### 5 Conclusions

The Matthews’ method predates computers and was designed for hand calculations. It is simple to use and understand, and is occasionally the basis of harvesting cost calculations in the literature. Also, the method is probably the preferred method by forest managers, forest owners or contractors when estimating harvesting costs or negotiating contracts.

The ASD-method often yields lengthy derivations and adaptations according to landing layout and parcel shape (e.g. Greulich 1989). Improving the method to include uneven terrain may be difficult or impossible in practice, at least there seems to be no attempts at this in the literature. However, once a formula is derived, it can be used in the same manner as the Matthews’ method. Also, cumbersome integral calculations can be estimated numerically (i.e. by Equation (9) on a grid of the parcel). This approach was suggested by Suddarth and Herrick (1964), who also found that dividing the parcel in eight parts yielded estimates deviating less than 1% from the ASD.

Another reason for using Matthews’ method or the ASD-method is that most of the productivity studies in the literature are reporting cycle times, suitable for the methods.

The network method is a promising method for the calculation of TTC. Once implemented, different parcel shapes can readily be calculated. In addition, the method is more flexible than the ASD-method. More complex TTC-functions \( f(x,y) \) in Equation (11) can be handled without affecting the computation time much.

### 5.1 Future research

All TTC models discussed here are simplifications of a real world problem, and thus incorrect. Barring Contreras and Chung (2007), who included a comparison of their method to the method of Greulich (1991) for some polygons, there are few publications discussing errors in the models or how well they describe the real world.

The network method has been applied to several forestry problems in the cited literature, even though studies correlating driving speed with e.g. micro topography and detailed inventories are lacking in the literature.

Hopefully, future productivity studies can shed light to which method is yielding better solutions for different problems, and also supplying better input data for the models.

### Acknowledgments

Thanks to the Norwegian Research Council (grant NFR186912/I30) for funding this study. Also thanks to two anonymous reviewers for their time and comments,
which improved the manuscript.

REFERENCES


